Solutions to JEE Main - 1 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $R=\sqrt{30^2+30^2+2\times30\times30\cos120^\circ}=30N$

$\tan\alpha=\dfrac{30\sin120^\circ}{30+30\cos120^\circ}=\sqrt{3}\ \Rightarrow\ \alpha=60^\circ$

$(\cos120^\circ=\cos(180^\circ-60^\circ)=-\cos60^\circ=-\dfrac{1}{2}$

$\&\ \sin120^\circ=\sin(180^\circ-60^\circ)=\sin60^\circ=\dfrac{\sqrt{3}}{2})$

2.(C) $\overline{r}=(-3\hat{i}-\hat{j}+2\hat{k})\ m$

$\overline{\tau}=\overline{r}\times\overline{F}$

$=(-3\hat{i}-\hat{j}+2\hat{k})\times(\hat{i}-2\hat{j}+\hat{k})=6\hat{k}+3\hat{j}+\hat{k}-\hat{i}+2\hat{j}+4\hat{i}=(3\hat{i}+5\hat{j}+7\hat{k})\ N.m$

3.(D) For three non collinear vectors adding up to zero, sum of magnitudes of 2 > magnitude of 3rd vector

(A) $2+3<8$

(B) $3+4<9$

(C) $5+6<20$

(D) $4+5>8$

4.(C) $\overline{a}=\dfrac{1}{4}(2\hat{i}-2\hat{j}+\hat{k})$ and $\dfrac{7}{4}\hat{i}-\dfrac{7}{4}\hat{j}+\dfrac{7}{8}\hat{k}=\dfrac{7}{8}(2\hat{i}-2\hat{j}+\hat{k})$

Hence, Option (C) is correct

5.(D) $AB\sin\theta=\sqrt{3}AB\cos\theta \quad \therefore \quad \tan\theta=\sqrt{3}$

$\theta=60^\circ \quad \therefore \quad |\overline{A}+\overline{B}|=\sqrt{A^2+B^2+2AB\cos60^\circ}=\sqrt{A^2+B^2+AB}$

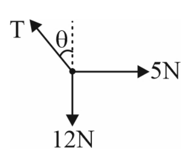
6.(C) $T\cos\theta=12$

$T\sin\theta=5$

Squaring & adding

$T^2=144+25=169$

$\Rightarrow \quad T=13N$



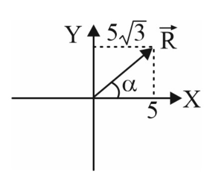
7.(A) $\overline{a}\perp\overline{a}\times\overline{b} \quad \Rightarrow \quad \overline{a}.(\overline{a}\times\overline{b})=0$

8.(B) Using law of vector addition:

$\vec{A}+\vec{B}+\vec{E}=0,\ \vec{B}+\vec{E}+\vec{D}=\vec{C}\ \&\ \vec{A}+\vec{C}=\vec{D}$

9.(A) $R_X=2+\sqrt{3}\times\dfrac{\sqrt{3}}{2}+5\times\dfrac{1}{2}-2\times\dfrac{1}{2}=5$

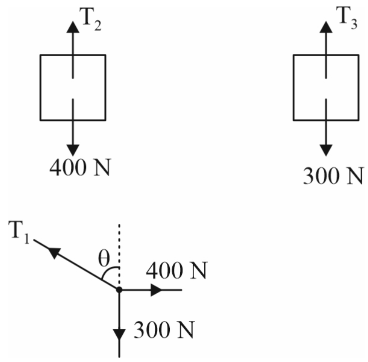
$R_y=\sqrt{3}\times\dfrac{1}{2}+5\times\dfrac{\sqrt{3}}{2}+\sqrt{3}+2\times\dfrac{\sqrt{3}}{2}=5\sqrt{3}$



$R = \sqrt{5^2 + (5\sqrt{3})^2} = 10N$

$\tan\alpha = \frac{5\sqrt{3}}{5} = \sqrt{3} \Rightarrow \alpha = 60°$

10.(B) $T_2 = 400N$



$T_3 = 300N$

$T_1 \sin\theta = 400$

$T_1 \cos\theta = 300$

$\Rightarrow \quad \tan\theta = \frac{4}{3} \quad \therefore \theta = 53°$

$T_1 \sin 53° = 400 \quad \therefore \quad T_1 = 500N$

Hence, Option (B) is correct

11.(C) $\bar{R} = \bar{P} + \bar{Q}$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ **... (i)**

$\bar{R}_1 = \bar{P} + 2\bar{Q}$

$R_1^2 = P^2 + 4Q^2 + 4PQ\cos\theta$

and $|\bar{R}_1| = 2|\bar{R}|$

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ … (ii)

$\bar{R}_2 = \bar{P} - \bar{Q}$

$R_2^2 = P^2 + Q^2 - 2PQ\cos\theta$ and $|\bar{R}_2| = 2|\bar{R}|$

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ ... (iii)

Solving (i), (ii) and (iii)

$|\vec{P}| : |\bar{Q}| = \sqrt{2} : \sqrt{3}$

12.(B) Projection of $\bar{P}$ along $\bar{Q} = \frac{\bar{P}.\bar{Q}}{Q}$

$= \frac{(2\hat{i} - \hat{j} + 2\hat{k})\cdot(3\hat{i} + 4\hat{j} + 12\hat{k})}{\sqrt{3^2 + 4^2 + 12^2}}$

$= \frac{6 - 4 + 24}{\sqrt{169}} = \frac{26}{13} = 2$

13.(D) Let Tension in the string be T; for the equilibrium of 2kg block :

$T = 2g = 20N$

For the equilibrium of 5 kg block, Net horizontal force will be zero :

$\Rightarrow \quad f = T\cos 45° = 20 \times \frac{1}{\sqrt{2}} = 10\sqrt{2}N$

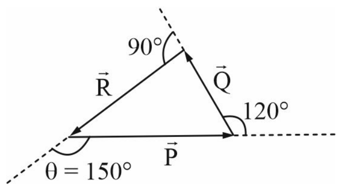
14.(B) As these vectors are orthogonal

$$\vec{A}.\vec{B}=0$$

Hence $\dfrac{2\hat{i}+\lambda\hat{j}+\hat{k}}{\sqrt{5+\lambda^2}}.\dfrac{\hat{i}-2\hat{j}+3\hat{k}}{\sqrt{14}}=0$

$$2-2\lambda+3=0; \qquad \lambda=\frac{5}{2}$$

15.(A) $\vec{P}, \vec{Q}, \vec{R}$ will be the sides of a triangle. Angle between $\vec{R}$ and $\vec{P}$, $\theta=150°$



16.(B) $\Delta\vec{r}=3\hat{i}+3\hat{j}+3\hat{k}$

$$\vec{F}=\frac{10}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$$

$$\vec{W}=\vec{F}\,.\,\Delta\vec{r}=\frac{10}{\sqrt{3}}\times(3+3-3)=10\sqrt{3}J$$

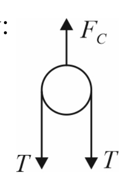
17.(C) $\cos\theta=\dfrac{\vec{a}\,.\,\vec{b}}{ab}=\dfrac{2\times1-1\times1+2\times1}{\sqrt{9}\sqrt{3}}=\dfrac{3}{3\sqrt{3}}=\dfrac{1}{\sqrt{3}}$

$$\Rightarrow \qquad \theta=\cos^{-1}\frac{1}{\sqrt{3}}$$

18.(C) Considering equilibrium of pulley:

$F_C=2T=24mg$

$=8mg$



19.(B) Given horizontal force $F=25N$ and the coefficient of friction between block and wall $(\mu)=0.4$.

We know that at equilibrium, horizontal force equals the normal reaction to the block against the wall.

Therefore, normal reaction to the block $(R)=F=25N$.

We also know that weight of the block (W) = Frictional force $=\mu R=0.4\times25=10\,N$

20.(B) Area of parallelogram $=\left|\vec{a}\times\vec{b}\right|=\left|4\hat{k}-5\hat{j}-4\hat{k}+10\hat{i}+4\hat{j}-8\hat{i}\right|=\left|2\hat{i}-\hat{j}\right|=\sqrt{5}m^2$

SECTION – 2

21.(60) $|\bar{a}|=1 \qquad |\bar{b}|=1$

$(\bar{a}+2\bar{b})\,.\,(5\bar{a}-4\bar{b})=0$

$5a^2-4ab\cos\theta+10ab\cos\theta-8b^2=0$

$5-4\cos\theta+10\cos\theta-8=0$

$6\cos\theta=3$

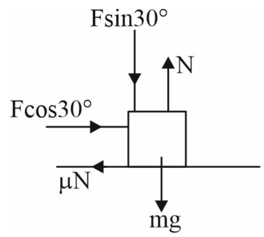
$\cos\theta = \frac{1}{2}$ $\quad\therefore\quad$ $\theta = 60°$

22.(60) $F\cos 30° = \mu N$ ... (i)

$N = F\sin 30° + mg$ ... (ii)

Solving (i) and (ii)

$F = 2mg = 60N$

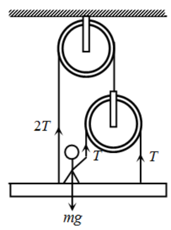


23.(15) Man : $T + N = mg$

Plank: $3T = N$

$\Rightarrow \quad 4T = mg$

$\therefore \quad T = \frac{60\times 10}{4} = 150$ N



24.(50) FBD of block:

$N + 50\sin 53° = W$

$\Rightarrow \quad N = W - 50\sin 53° = 80 - 50\times\frac{4}{5} = 40N$

Also $f = 50\cos 53° = 50\times\frac{3}{5} = 30N$

$F_{\text{net, ground}} = \sqrt{N^2 + f^2} = \sqrt{30^2 + 40^2} = 50N$

25.(10) Let $\quad \vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{B} = 3\hat{i} + 4\hat{j}$

If $\quad \vec{R} = \vec{A} + \vec{B}$, given that $R_x = 0 \quad\Rightarrow\quad A_x = -3$

Also $\quad R_y = 4 - A_y = |\vec{B}| \quad\Rightarrow\quad 4 + A_y = 5 \quad\Rightarrow\quad A_y = +1$

$\Rightarrow \quad |\vec{A}| = \sqrt{3^2 + 1^2} = \sqrt{10}$

CHEMISTRY

SECTION-1

1.(C) meq. of acid $= 100 \times 0.2 \times 2 = 40$

meq. of NaOH $= 100 \times 0.2 = 20$

meq. of excess acid $= 40 - 20 = 20$

$N_{mix} = \frac{20}{200} = 0.1\,N$

2.(C) **i.** 1 molecule of $O_2 = \frac{32}{6 \times 10^{23}}$ **ii.** 1 atom of $N = \frac{14}{6 \times 10^{23}}$

iii. 1 mol of $H_2O = 18\,g$ **iv.** Weight of $Fe = 10^{-10}\,g$

3.(B) $CaCO_3$ will react with H_2SO_4

meq. of $H_2SO_4 =$ meq. of $CaCO_3$

$= \frac{1}{15} \times 2 \times 30 = 4$

meq. of $CaCO_3 = 4 = \frac{g}{E} \times 1000$

Mass of $CaCO_3(g) = \frac{4 \times E}{1000} = \frac{4 \times 50}{1000} = 0.2\,g$

Mass of $NaCl = 1 - 0.2 = 0.8\,g$

% of $NaCl = 80\%$

4.(A) Volume of 1 drop of $H_2O = 0.04\,mL$

Weight of 1 drop of H_2O = Volume × Density $= 0.04 \times 1 = 0.04\,g$

1 mole of $H_2O = 18\,g = 6.023 \times 10^{23}$ molecules

$\therefore \quad \frac{6.023 \times 10^{23} \times 0.04}{18} = 1.3384 \times 10^{21}$ molecules

5.(B) meq. of H_3PO_3 = meq. of KOH

$M_a \times n_f \times V_a = M_b \times V_b \times n_f$

$0.1 \times 2 \times 20 = 0.2 \times 1 \times V_b \quad \Rightarrow \quad V = 20\,mL$

6.(A) Equation of dilution :

$N_1V_1 = N_2V_2$

$\frac{1}{4} \times V_1 = \frac{1}{10} \times 1000 \qquad V_1 = 400\,mL$

Volume of H_2O added $= 1000 - 400 = 600\,mL$

7.(A) $N_2 + 3H_2 \longrightarrow 2NH_3$

$n_{H_2} = \frac{5}{2} = 2.5 \qquad n_{N_2} = \frac{14}{28} = 0.5$

1 mole of N_2 reacts with 3 moles of H_2.

0.5 mole of N_2 reacts with 1.5 moles of H_2.

$\Rightarrow \quad n_{H_2}$ unreacted $= 2.5 - 1.5 = 1\,mole \quad \Rightarrow \quad 2g$

8.(C) Final conc. of H^+ ions $= \dfrac{N_1V_1 + N_2V_2}{V_{total}} = \dfrac{(100\times 0.3)+(100\times 0.3)}{200} = \dfrac{30+30}{200} = \dfrac{60}{200} = \dfrac{3}{10} = 0.3\,N$

9.(A) As per the reaction, theoretical yield of the Ti is 1.88 moles and actual yield is $\dfrac{2}{3}$ moles.

$\therefore$ % yield of Ti $= \dfrac{2/3}{1.88}\times 100 = 35.46\%$

10.(B) $NaI + AgNO_3 \longrightarrow AgI + NaNO_3$

x mole (AgNO₃) x mole (AgI)

$2AgI + Fe \longrightarrow FeI_2 + 2Ag$

x mole (AgI) x/2 mole (FeI_2)

$2FeI_2 + 3Cl_2 \longrightarrow 2FeCl_3 + 2I_2$

x/2 mole (FeI_2) x/2 mole (I_2)

$\dfrac{x}{2}\text{mole} = \dfrac{254\times 10^3}{254} = 1000$

$\Rightarrow$ x = 2000 moles, $W_{AgNO_3} = 34\times 10^4\,g$

11.(A) $CaCO_3 + 2HCl \longrightarrow CaCl_2 + H_2O + CO_2$

n = $\dfrac{20}{100}$ $\dfrac{20}{36.5}$

0.2 0.54

Here limiting reagent is $CaCO_3$

1 mole $CaCO_3 =$ 1 mole CO_2

0.2 mole $CaCO_3 =$ 0.2 mole CO_2

Mass $= n\times M^\circ = 0.2\times 44 = 8.8\,g$

12.(D) Molality = 0.2

$\Rightarrow$ 0.2 moles H_2SO_4 in 1000 g solvent

Mass of $H_2SO_4 = 0.2\times 98 = 19.6\,g$

Mass of solvent $= 1000\,g$

Total mass of solution $= 19.6 + 1000 = 1019.6\,g$

13.(A) **I.** Molality and mole fraction are independent of small change in temperature, in which there will no evaporation losses.

II. Not correct : Molar volume of ideal gases is 22.4 L only at NTP/STP.

14.(B) $MgCO_3 \longrightarrow MgO + CO_2$

8 g MgO is formed

$n_{MgO} = \dfrac{8}{40} = \dfrac{1}{5}$ mole MgO is formed

1 mole $MgO \equiv 1$ mole $MgCO_3$

$\therefore$ Moles of $MgCO_3$ used $= \dfrac{1}{5}$

Mass of $MgCO_3$ used $= \dfrac{1}{5}\times 84 = 16.8$

% purity $= \dfrac{16.8}{20}\times 100 = 84\%$

15.(D) $\left\{z = \frac{x(z-2)}{100} + \frac{(100-x)(z+1)}{100}\right\}$

$\Rightarrow \quad 100z = xz - 2x + 100z + 100 - xz \quad \Rightarrow \quad x = \frac{100}{3}$ or $x = 33.33\%$

16.(D) $AgNO_3 \xrightarrow{\Delta} Ag(s) + NO_2(g) + \frac{1}{2}O_2(g)$

$n = 0.4$	–	–	–
–	0.4	0.4	0.2

Moles of gases obtained = 0.6

$\Rightarrow$ Volume of gases $= (0.6 \times 22.4)\,L = 13.44\,L$

17.(D) Reactions involved $Na_2CO_3 + HCl \longrightarrow NaHCO_3 + NaCl$

m moles of HCl used = m moles of Na_2CO_3

$1 \times V = \frac{1}{106} \times 1000; \quad \therefore \quad V = 9.43\,mL$

18.(B) Chalk $(CaCO_3)$

Let mass of $CaCO_3$ in mixture $= x\,g$

$CaSO_4$ does not react with H_2SO_4 as it is neutral solution in aqueous medium.

Only $CaCO_3$ react with acid.

Excess of H_2SO_4 is neutralised by $Al(OH)_3$

meq. of $Al(OH)_3 =$ meq. of excess $H_2SO_4 = 310 \times \frac{1}{10} \times 3 = 31 \times 3 = 93$

meq. of H_2SO_4 taken $= 290 \times \frac{1}{5} \times 2 = 116$

meq. of H_2SO_4 used for $CaCO_3 = 116 - 93 = 23$

meq. of $CaCO_3 = 23 = \frac{g}{E} \times 1000$

$g_{CaCO_3} = \frac{23 \times 50}{1000} = 1.15\,g\,; \qquad \%\,CaCO_3 = \frac{1.15}{2} \times 100 = 57.5\%$

19.(B) The volume of NaOH used is 5 mL as given in the question

meq. of NaOH = meq. of oxalic acid

$M \times 5 = 1.25 \times 2 \times 10;\; M = \frac{1.25 \times 2 \times 10}{5} = \frac{25}{5} = 5M$

So the answer will be 5.

20.(C) Let mmol of NaOH = x, $Na_2CO_3 = y$

Reaction in presence of phenolphthalein indicator.

meq. of HCl = meq. of NaOH + $\frac{1}{2}$ meq. of Na_2CO_3

$20 \times 1 = x + \frac{y}{2} \times 2$

Reactions in presence of methyl orange

meq. of HCl $= \frac{1}{2}$ meq. of Na_2CO_3

$5 \times 1 = \frac{y}{2} \times 2 \quad \Rightarrow \quad y = 5;\; x = 15$

SECTION - 2

21.(8) Molar mass of $Na_2SO_4 \cdot nH_2O = (142 + 18n)$

% Loss in mass $= \dfrac{18n}{(142+18n)} \times 100$

$50.3 = \dfrac{18n \times 100}{142 + 18n} \quad \Rightarrow \quad n = 8$

22.(8) 100 g of protein has 0.07 g of Fe.

So, 56 g of Fe should have minimum weight of $\dfrac{100}{0.07} \times 56 = 80000 = 8 \times 10^4$ g

23.(10) 90% w/w alcohol

$d = 0.8\,g/mL$

Molarity $= M_1 = \dfrac{10xd}{M°}$

$M_1 = \dfrac{10 \times 90 \times 0.8}{M°} = \dfrac{720}{M°}$

80 mL 10% w/w alcohol $(d = 0.9\,g/mL)$

$M_2 =$ Molarity $= \dfrac{10xd}{M°} = \dfrac{10 \times 10 \times 0.9}{M°} = \dfrac{90}{M°}$

Equation of dilution :

$M_1V_1 = M_2M_2$

$\dfrac{720}{M°} \times V_1 = \dfrac{80 \times 90}{M°} \quad \Rightarrow \quad V_1 = \dfrac{80 \times 90}{720} = 10\,mL$

24.(3) meq. of acid = meq. of base

$10 \times 0.1 \times n_f = 30 \times 0.05 \times 2$

$n_f = \dfrac{30 \times 0.05 \times 2}{10 \times 0.1} = 3$

25.(3) Molarity $= \dfrac{10xd}{M°} = \dfrac{10 \times 30 \times 0.365}{36.5} = 3$ molar

MATHEMATICS

SECTION-1

1.(A) We have $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$; $\Rightarrow \alpha$ and β are roots of equation $x^2 = 5x - 3$ or $x^2 - 5x + 3 = 0$

$\therefore \quad \alpha + \beta = 5$ and $\alpha\beta = 3$

Thus, the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0 \Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $\quad 3x^2 - 19x + 3 = 0$

2.(A) $\frac{x+7}{x-5} + \frac{3x+1}{2} \geq 0$

$\Rightarrow \quad \frac{2x+14+(x-5)(3x+1)}{2(x-5)} \geq 0 \quad \Rightarrow \quad \frac{2x+14+3x^2+x-15x-5}{(x-5)} \geq 0$

$\Rightarrow \quad \frac{3x^2-12x+9}{(x-5)} \geq 0 \quad \Rightarrow \quad \frac{x^2-4x+3}{(x-5)} \geq 0 \quad \Rightarrow \quad \frac{x^2-3x-x+3}{(x-5)} \geq 0$

$\Rightarrow \quad \frac{(x-3)(x-1)}{(x-5)} \geq 0 \quad \Rightarrow \quad x \in [1, 3] \cup [5, \infty)$ but $x \neq 5 \quad \Rightarrow \quad x \in [1, 3] \cup (5, \infty)$

3.(D)

4.(A) The inequality is $|x+2| - |x-2| < x - \frac{3}{2}$

Dividing the problem into three intervals:

(i) If $x < -2$, then $-(x+2) + (x-1) < x - \frac{3}{2}$

$\Rightarrow \quad x > -\frac{3}{2}$

But $-\frac{3}{2} > -2$, hence no common values

(ii) If $-2 \leq x < 1$, then $(x+2) + (x-1) < x - \frac{3}{2} \Rightarrow x < -\frac{5}{2}$

But $-\frac{5}{2} < -2$, hence no common values

(iii) If $x \geq 1$, then

$\Rightarrow \quad x > \frac{9}{2}$

$\because \quad \frac{9}{2} > 1$

$\Rightarrow$ common solution is $x > \frac{9}{2} \Rightarrow x \in \left(\frac{9}{2}, \infty\right)$

$\therefore$ Solution set is $x \in \left(\frac{9}{2}, \infty\right)$

5.(C) $\frac{x^2+6x-7}{|x+4|} < 0$

$\Rightarrow$ $x^2+6x-7<0$, provided $x+4 \neq 0$

$[\because |x+4| > 0 \text{ if } x \neq -4]$

$\Rightarrow$ $(x+7)(x-1)<0,\ x \neq -4 \Rightarrow -7<x<1$

$x \neq -4$

$\therefore$ $x \in (-7, -4) \cup (-4, 1)$

6.(C) Since $\frac{x^2+x}{x+4} > 0$ and $0 < 0.8 < 1$

$\therefore$ The given inequality implies $log_6\left(\frac{x^2+x}{x+4}\right) > 1$

$\Rightarrow$ $\frac{x^2+x}{x+4} > 6 \Rightarrow \frac{x^2-6x+x-24}{x+4} > 0 \Rightarrow x \in (-4, -3) \cup (8, \infty)$

7.(A) $175 = 5^2.7,\ 245 = 5.7^2,\ 875 = 5^3.7,\ 1715 = 5.7^3$

Let $\alpha = log\,5,\ \beta = log\,7$

$$a = \frac{log\,175}{log\,245} = \frac{2\alpha+\beta}{\alpha+2\beta}$$

$$b = \frac{log\,875}{log\,1715} = \frac{3\alpha+\beta}{\alpha+3\beta}$$

$$\frac{1-ab}{a-b} = \frac{(\alpha+2\beta)(\alpha+3\beta)-(2\alpha+\beta)(3\alpha+\beta)}{(2\alpha+\beta)(\alpha+3\beta)-(\alpha+2\beta)(3\alpha+\beta)} = \frac{5(\beta^2-\alpha^2)}{\beta^2-\alpha^2} = 5$$

8.(A) **I.** $-75 < 3x-6 \Rightarrow x > -23$

$3x-6 \le 0 \Rightarrow x \le 2$

II. $14 \le 3x+11 \Rightarrow 3 \le 3x \Rightarrow 1 \le x$

$3x+11 \le 22 \Rightarrow 3x \le 11 \Rightarrow x \le \frac{11}{3}$

III. $-20 \le 2-3x \Rightarrow x \le \frac{22}{3}$

$2-3x \le 36 \Rightarrow -34 \le 3x \Rightarrow x \ge \frac{-34}{3}$

9.(A) Let $a\frac{(x-b)(x-c)}{(a-b)(a-c)} + b\frac{(x-c)(x-a)}{(b-c)(b-a)} + c\frac{(x-a)(x-b)}{(c-a)(c-b)} - x$ s

We have $f(a)=0,\ f(b)=0$ and $f(c)=0$

$f(x)$ is quadratic equation having more than 2 roots.

Thus, $f(x) \equiv 0$

10.(B) $\alpha + \beta = -2|a|$ and $\alpha \cdot \beta = 4$

α	2	4	–2	–4
β	2	1	–2	–1

$\Rightarrow \quad a = \frac{-5}{2}$

11.(C) $x^2 + ax + c = (x-\gamma)(x-\delta)$

Thus, $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} = \frac{\alpha^2 + a\alpha + c}{\beta^2 + a\beta + c}$

$= \frac{-b+c}{-b+c} = 1 \quad [\because \alpha, \beta$ are roots of $x^2 + ax + b = 0]$

12.(A) $\frac{c}{a} < 0$

$\Rightarrow \quad a^2 - 2a < 0$

$\Rightarrow \quad 0 < a < 2$

13.(C) Let $y = \frac{x^2 - bc}{2x - b - c}$

$\Rightarrow \quad x^2 - 2yx + (b+c)y - bc = 0$

$\because \quad x \in R$ so $4y^2 - 4(b+c)y + 4bc \geq 0$

$\Rightarrow \quad y \leq b$ or $y \geq c \quad (\because b < c)$

14.(A) $ax^2 - 2bx + c = 0$

$\alpha + \beta = -\frac{(-2b)}{a} = \frac{2b}{a}$

$\alpha\beta = \frac{c}{a}$

$\alpha^3\beta^3 + \alpha^2\beta^2 + \alpha^3\beta^2$

$= \alpha^2\beta^2(\alpha\beta + \alpha + \beta) = (\alpha\beta)^2(\alpha\beta + \alpha + \beta)$

$= \left(\frac{c}{a}\right)^2\left(\frac{c}{a} + \frac{2b}{a}\right) = \frac{c^2}{a^2} \times \left(\frac{(c+2b)}{a}\right) = \frac{c^2.(c+2b)}{a^3}$

15.(C) Given $a + b = -a,\ ab = b$

As $b \neq 0$, we get $a = 1$

Thus, $x^2 + ax + b = x^2 + x - 2 = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4} \geq -\frac{9}{4}$

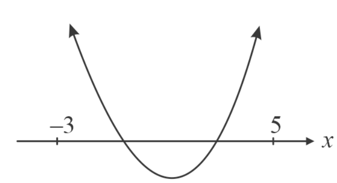
$\therefore$ Least value of $x^2 + ax + b$ is $-\frac{9}{4}$ which is attained at $x = -\frac{1}{2}$

16.(D) Let $f(x) = x^2 - 2kx + k^2 = 4$

Conditions are:

(1) $D \geq 0 \quad \Rightarrow \quad (-2k)^2 - 4(k^2 - 4) = 16 \geq 0\ \forall\ k$

(2) $f(-3) > 0 \quad \Rightarrow \quad k^2 + 6k + 5 > 0 \ \Rightarrow\ k < -5$ or $k > -1$



(3) $f(5)>0$ $\Rightarrow$ $k^2-10+21>0 \Rightarrow k<3$ or $k>7$

(4) $-3<k<5$

$\therefore$ Values of k satisfying conditions (1), (2), (3), (4) are:

$-1<k<3$

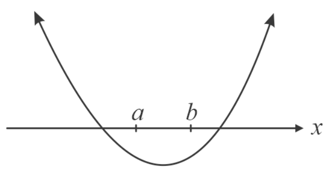
17.(B) Let the discriminant of the equation $x^2+px+q=0$ be D_1 then $D_1=p^2-4q$ and the discriminant D_2 of the equation $x^2+rx+s=0$ is $D_2=r^2-4s$

$\therefore$ $D_1+D_2=p^2+r^2-4(q+s)=p^2+r^2-2pr$ [From the given relation]

$\therefore$ $D_1+D_2=(p-r)^2\geq 0$

Clearly at least one of D_1 and D_2 must be non-negative consequently at least one of the equation has real roots.

18.(C) Let $f(x)=(x-a)(x-b)-c \Rightarrow f(a)=-c=f(b)<0$



19.(B) Given $\alpha<\beta, c<0, b>0$

$\therefore$ $\alpha+\beta=-b<0$ and $\alpha\beta=c<0$

Clearly, α and β have opposite signs and $\alpha<\beta$

$\therefore$ $\alpha<0$ and $\beta>0 \Rightarrow \alpha<0<\beta$

Further $\alpha+\beta<0 \Rightarrow \beta<-\alpha \Rightarrow |\beta|<|-\alpha|$

$\Rightarrow$ $\beta<|\alpha|$

Hence, $\alpha<0<\beta<|\alpha|$

20.(A) $ax^2+bx+c=0\rightarrow$ roots α,β **…(i)**

$\ell x^2+mx+n=0\rightarrow$ roots $\frac{1}{\alpha},\frac{1}{\beta}$

$\Rightarrow$ $\ell\left(\frac{1}{x}\right)^2+m\left(\frac{1}{x}\right)+n=0\rightarrow$ roots α,β

$\Rightarrow$ $nx^2+mx+\ell=0\rightarrow$ roots α,β **...(ii)**

From (i) and (ii)

$a:b:c=n:m:\ell$

SECTION - 2

21.(3) Let α, α^2 be the roots of $3x^2+px+3=0$

Now, $S=\alpha+\alpha^2=-\frac{p}{3}$,

$P=\alpha^3=1$

$\left(\alpha+\alpha^2\right)^3=\left(\frac{-p}{3}\right)^3$

$\Rightarrow \quad \alpha^3+\left(\alpha^3\right)^2+3\alpha^3\left(\alpha+\alpha^2\right)=\left(\frac{-p}{3}\right)^3$

$\Rightarrow \quad 1+1-p=\left(\frac{-p}{3}\right)^3 \quad \Rightarrow \quad p=3$

22.(15) If α is a common root of the three equations, then $\alpha^2+a\alpha+12=0,\ \alpha^2+b\alpha+15=0$

and $\quad \alpha^2+(a+b)\alpha+36=0$

$\Rightarrow \quad b\alpha=-24,\ a\alpha=-21$

Thus, $\quad \alpha^2-21+12=0 \Rightarrow \alpha=\pm 3$

As $\alpha>0$, we get $\alpha=3$

Therefore, $a=-7,\ b=-8$

23.(69) $A_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$aA_n-bA_{n-1}=a\left(\alpha^n+\beta^n\right)-b\left(\alpha^{n-1}+\beta^{n-1}\right)$

Now, $\quad \alpha+\beta=\underline{a},\ \alpha\beta=b \qquad \therefore \qquad aA_n-bA_{n-1}=(\alpha+\beta)\left(\alpha^n+\beta^n\right)-\alpha\beta\left(\alpha^{n-1}+\beta^{n-1}\right)$

$=\alpha^{n+1}+\alpha\beta^n+\alpha^n\beta+\beta^{n+1}-\alpha^n\beta-\alpha\beta^n=\alpha^{n+1}+\beta^{n+1}=A_{n+1}$

$\therefore \quad A_{n+1}=aA_n-bA_{n-1}$

24.(1) Let $log_{16}x=t$

Hence, $t^2-t+log_{16}k=0$ and $k>0$

The equation has exactly one solution. Hence, the discriminant must be zero.

$\Delta=1-4\,log_{16}k$

$\Rightarrow \quad 0=1-4\,log_{16}k$

$\Rightarrow \quad log_{16}k=\frac{1}{4}$

$\Rightarrow \quad k=2$

25.(2) Given equations are $x^2-3x+a=0$ **. . .. (i)**

and $\quad x^2+ax-3=0$ **. . .. (ii)**

Subtracting (ii) from (i), we get:

$-3x-ax+a+3=0$

$\Rightarrow \quad (a+3)(-x+1)=0 \quad \Rightarrow \quad$ either $a=-3$ or $x=1$

when $a=-3$, the two equations are identical.

So, we taken = 1, which is the common roots of the two equations. Substituting x = 1 in (i), we get:

$1+a-3=0 \quad \Rightarrow \quad a=2$